A PRACTICAL WAY TO MAINTAIN QUALITY WHEN AND HOW TO REDUCE SPC SAMPLE SIZE

William J. Latzko, Ph.D.
North Bergen, NJ
wjl@Latzko,us

SUMMARY

Many books discuss the mechanics of initially setting up a method for measuring processes often called Statistical Process Control (SPC). A number of these books show how to identify special causes of variation that impact the process quality. Often, advice is presented of how to improve the process once special causes are identified. Some of the books also discuss important issues such as rational sub-grouping. However, few texts discuss what Shewhart called "control charts as an operation" once the special causes are identified and acted upon.

This paper will show the importance of setting up the initial study properly and explain how to reduce the sampling procedure to minimize costs. It will also present tables for the factors to use in reducing the sample or subgroup size. It applies to manufacturing and non-manufacturing processes.

INTRODUCTION

Ever since Dr. Walter Shewhart (1931) published his book, *The Economic Control of Quality of Manufactured Product*, the world has been aware of the method of Statistical Quality Control (SQC). Although many texts tell us how to set up a process measurement chart (also called control chart), these texts say little about what Woodall(2000) calls Phase 2. This phase is the use of the chart to monitor the process as an ongoing operation.

Palm(2000) distinguishes Phase 1 in three stages: in stage A we end up with "with control limits with which we can begin effective real time charting." In stage B we improve the process. Palm points out that stage B will remove such special (assignable) causes as we can control and that are economical to remove. For practical purposes, the process functioning satisfactorily even though some special cause may recur while some rare special causes are too costly to remove. We wish to guard against the impact of these special causes recurring. In stage C we monitor processes in an ongoing basis to detect any special causes that require management action.

Wheeler and Chambers (1992) show a flowchart that gives the "logic behind control charts." They use five stages the first four of which are Phase 1 and the last Phase 2. Later in their book they present a flowchart of "Using Shewhart Charts for Continual Improvement." Deming (1992) quoting Shewhart, states that there are two uses for control charts, "as a judgment" and as an operation (ongoing)." His text clearly indicates that Shewhart was talking about Phase 1 and Phase 2 respectively.

Apparently, everyone agrees that the Phase 1 use of a control chart is to remove special causes of variation, those forces that cause the quality to be inconsistent. There is also agreement to extend the control limits resulting from Phase 1 to Phase 2. There is disagreement among statisticians about the interpretation of those control limits. However, their practical use is not in question.

Wheeler (1992) uses Chaos Theory to show that the force of Entropy is continually driving processes from the ideal state reached in Phase 2 back to the chaotic state of Phase 1. This theory matched the author's experience in both manufacturing and non-manufacturing operations. In fact, the author's experience is that the force of entropy works faster in service areas due to the human factors involved. This is a reason why phase 2 charts are a necessity.

Since in Phase 1 efforts one knows little about the process, one uses as frequent a sample and reasonable sample sizes to detect and remove the special cause. Phase 2 is a monitoring mode where one has statistical data about the process. As a result, it is possible to reduce the amount of measurement.

According to Deming (1992, p. 337) Shewhart distinguished between two basic uses of control charts: 1) as a judgment and 2) as an operation (ongoing).

When the control chart is used as a judgment "we look at a control chart to observe whether the process that made a particular batch of product was in statistical control. If yes, then we know, for the quality-characteristic that was plotted on the chart, the distribution of this quality characteristic for individual items."(Deming, 1992)

When the control chart is used as an ongoing operation, Deming (1992, p. 337) says

> A control chart can also be used to attain and maintain statistical control during production. Here the process has already been brought into statistical control (or nearly so, with only rare evidence of a special cause). We extend into the future the control limits on (e.g.) an $\overline{x}$-chart , and plot points one by one, perhaps every half-hour or every hour. The up and down movements of the points are to be disregarded by the production worker unless they show a run (as for wear of tool), or unless a point falls outside the control limits.

This paper deals with control charts used for ongoing operations (Woodall's Phase 2 and Palm's Stage C), primarily for process adjustment and extended monitoring.

CONTROL CHARTS

The use of a control chart is to determine whether the observed variation is from a chance or common cause of variation or if it is due to an assignable or special cause. If the latter is the case then corrective action should be taken, the special cause identified, removed, and, if possible, prevented from recurring. If only chance or common causes are present and they are not delivering satisfactory product or service, management must change the process so that it will meet the customer's needs.

The word "control" has a number of interpretations. One interpretation (The Random House College Dictionary, 1975) is "to exercise restraint or direction over; dominate, command". In using the words "control chart", Shewhart (1931, p. 6) had a different meaning in mind when he defined control as: "for our present purpose a phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon may be expected to vary in the future."

Shewhart's definition of control, namely the ability to predict that a stable process will continue to operate within given limits in the future implies that he thought of the control chart as a device to monitor ongoing processes.

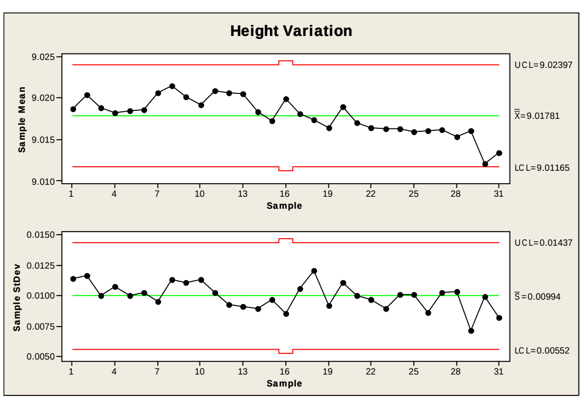

Figure 1. Example of Control Chart

The Shewhart control chart is, in fact, an operational definition of an assignable or special cause of variation. If the control chart of a stable process (one without special causes of variation) exhibits unstable patterns, one can suspect that special or assignable causes are present. It is certainly very worthwhile to examine the process for such effects.

For example, in measuring the height of a product it was observed that the first 13 averages were above the mean while the last 11 averages were below the mean. This indicates a significant change took place.

On investigating it was found that the process was changed at point 15 (coinciding with the end of a shift).

Left alone, the process would have continued well in control. In fact, the C_{pk} was 1.47 for the tampered process indicating that the original process was not only in control but well able to meet the requirements of the customer. As luck would have it, even with the tampering, the process was robust enough to yield good product. Normally one is not that fortunate.

Professor Burr has often been quoted that the control chart is the process talking to us. In one of his books (Burr, 1976, p. 60) he states, "In the initial stages of a control-chart application, we are primarily concerned with letting the process do the talking, that is, we collect the data, plot them, make the calculations, draw the lines, and interpret the results."

The process is any activity or set of activities that produces an output. The outcome can be either a product or service. In either case, the process output may be complete or not. For example, in taking an order in an e-business, one may need to check the customer's credit. The process of credit check produces an output that the customer's credit is either sufficient or not. This output now becomes the input to another process. If the credit was sufficient, the order process is completed. If the credit was not sufficient, alternative processes begin.

Many processes are not critical. Failure in these processes, while not desirable are not vital and will do little to harm the organization. Other processes are critical. It pays the organization to use a process control chart in such situation. Methods for determining whether a process should be monitored with a control chart have been described by elsewhere (Latzko, 1987). Such methods help to decide whether a particular process should be monitored or not. If the decision is to monitor the process one follows Burr's advice and sets up data collection to produce a suitable control chart. Once the chart is analyzed, one can determine whether assignable or special causes exist. If they exist, they are removed or neutralized. From this initial effort one can determine whether it pays to continue to collect data and at what level of activity. Thus such a preliminary system determines whether we created a chart for judgment or an ongoing chart.

In any case, it is critical that the control chart be prepared properly. This means that the concept of rational subgroup is vital to the success in using the chart and getting useable results, results that allow management to make vital decisions without being misled by poor data.

RATIONAL SUBGROUPS

Shewhart in both of his books (1931) (1939) stresses the importance of using rational subgroups. There are four issues raised when considering rational subgroups:

- What should be the nature of the subgroups
- How big a sample size should be used for each subgroup
- How frequently do we take samples during a day, week, month, etc.,
- How many subgroups should be sampled before looking whether special causes exist

Failure to consider the implications of these four areas can lead to control charts that give the wrong or misleading information. The bias caused by this is not easily detected. The four areas need to be considered before undertaking any data collection, measurement, or analysis.

Nature of Subgroups

In almost all books on control charts, the homogeneous nature of the subgroups is emphasized. It has been the writer's experience that this factor is frequently neglected or ignored. Often this is the case because someone deems it too hard (or expensive) to collect the data properly. The result is that special causes remain hidden and, thereby, deviling the process. The purpose of using a homogeneous subgroup is to reduce the variation *within* subgroups so that one can observe the variation *between* subgroups.

Shewhart discusses the advantages of what he calls "classification" in problem solving. If one deals with a mix of cause systems, even a signal that something was wrong would involve untangling that system to find the cause of failure. It is much easier to look at each component separately and treat each individually. In marketing theory, market segmentation is a well accepted process. In the financial world, classification is the essence. It is also needed in quality control. As Shewhart (1931, p. 299) states, "Obviously, the ultimate object is not only to detect trouble but also to find it, and such discovery naturally involves classification. The engineer who is successful in dividing his data initially into rational subgroups based upon rational hypotheses is therefore inherently better off in the long run than the one who is not thus successful."

If several people in an office carry out the same activity one can do more in improving the work by measuring the performance of each individual than the combined output. For more details see (Latzko, 1975, 1985). Another common illustration is that of a machine which produces multiple parts from individual components. For example, a plastic extrusion machine making bottles by injecting resin into 24 molds. Measuring the combined output of the 24 molds may lead one to think that the machine has no special causes present when in fact one or more of the molds may be so different as to increase the variability, causing the control limits to be spread farther apart than they should be.

The following is an example of the impact of mixing streams of work is based on Shewhart's discussion of maximum control (Shewhart, 1931, p. 158 ff.) Two pieces of equipment were measured individually. The distributions of each were nearly normal but the means were about 1.5 standard deviations apart.

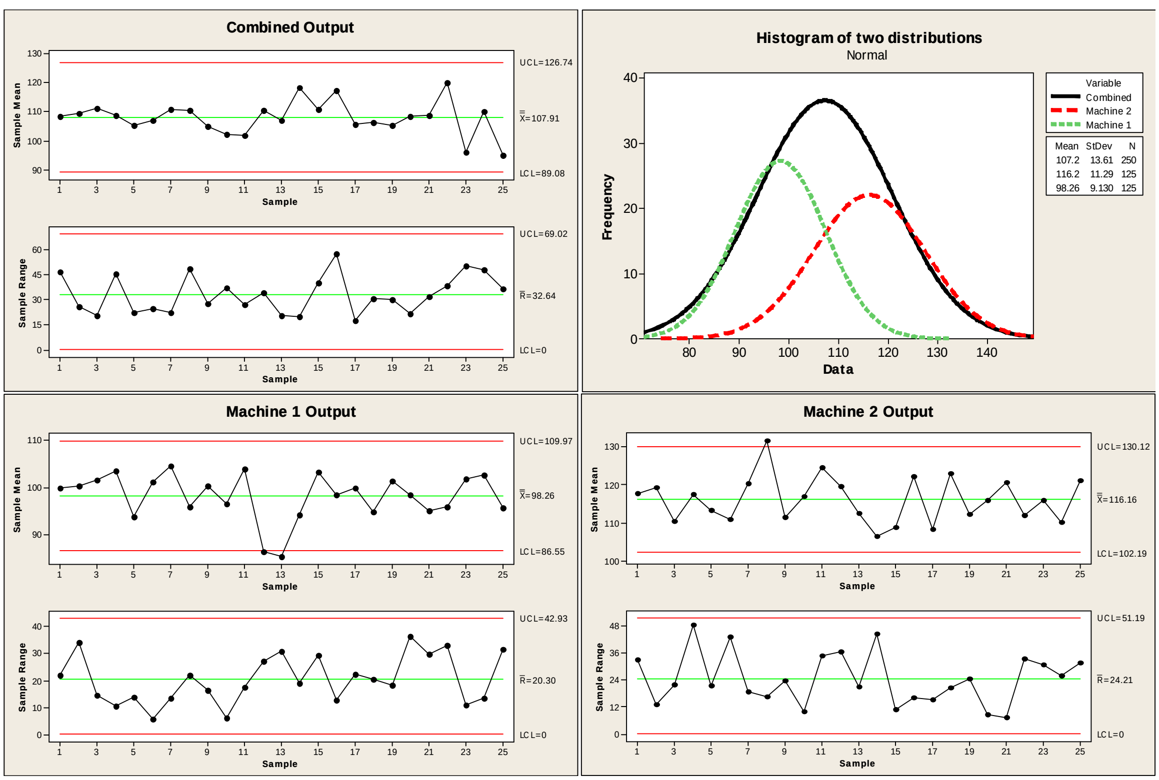

Figure 2. Non Homogeneity of Output

Below is a control chart of the combined output, each machine and the distributions:

Figure 2 shows that while each distribution is nearly normal, combining them (upper left) indicates no special causes are present. Yet, the control chart of each individual machine (bottom row) shows that an assignable or special cause of variation exists in each machine. The reason is that the variation of the combined output is about $\sqrt{2} = 1.414$ as large as the individual variation since $\sigma_{combined} = \sqrt{\sigma^2_{mach.1} + \sigma^2_{mach.2}}$ assuming no correlation effect present. Homogeneity is important!

Subgroup Size

Larger is not better. The inventor of control charts, Dr. Walter A. Shewhart, was a practical man. He had set out to solve the practical problem of improving the quality of telephones. He realized that statistics could help. With theory and a great deal of experimentation he arrived at the conclusion that the control chart, which is based on a pragmatic model, gives him the most economical way of avoiding two mistakes: 1) leaving the process alone when he should have intervened and 2) intervening in the process when he should leave it alone. In the 80 plus years that followed his brilliant concept—his chart was first described in a memorandum dated May 16, 1924—his method has stood up very well to the test of time.

He chose the subgroup size of four although he himself stated, " A little consideration will show that there is nothing sacred about the number four although there are several reasons why it may be the most satisfactory when there is no *a priori* knowledge to justify any other sample size."(Shewhart, 1931, p. 313) Shewhart continues saying that any small number would do. However he excludes a subgroup size of one since one would not be able to compute the standards deviation of a subgroup of 1. We have since tried to overcome this limitation by using chart for individuals with a moving range. One can only wonder if Shewhart would consider this as a proper control chart of the type he envisioned:

> In fact, the sensitivity of the test will increase, in general, with decrease in subsample size until the size of the sample is such that the data in any given subgroup come from a constant system of chance causes. In the absence of any *a priori* information making it possible to divide the data into rational subgroups, there would be some advantage therefore in reducing the subsample size to unity. To do so, however, would obviously defeat our purpose since we could not then obtain an estimate σ to use in the control charts. Hence we must choose some subsample size greater than unity. Sizes 2 and 3 offer some difficulties in the way of computation of σ and so we go to a sample of four. (Shewhart, 1931, p. 314)

There were three points that Shewhart made as far as sample size was concerned: 1) make it as small as possible other than unity because it is less likely to have assignable causes within a small sample, 2) use size 4 since it is easier to compute, and 3) if you know something about the process, a bigger size might do better.

Frequency of sampling

Shewhart made the point that small samples taken more frequently are better than larger samples taken at less frequent intervals. Duncan (1974, p. 447) questions this concept. Duncan maintains that it depends on the process. He gives several scenarios that might lead one to think that a larger sample size would spot a problem sooner. However, in the absence of such specialized knowledge he also suggests the use of a small sample size. The writer tries to use the pragmatic approach by using a small sample at intervals as frequent as one can economically stand until more is known about the behavior of the process. At that point, a better decision as to sampling frequency can be made.

Wheeler (2002, p. 236) distinguishes between the case which he calls "Regular Control Chart Data" and Periodically Collected Data". In the former the size and frequency can be independent of one another. This can consist of something like "high volume production of discrete parts". In the other case consisting of such things as "monthly summaries of business activities, samples from finished batches," etc., the measurement and time period are related. The larger the sample size in such a case, the longer one has to wait for the results of the analysis. Frequent smaller samples (perhaps weekly or daily business results) are preferred to get results to management as quickly as possible.

Number of Subgroups

The number of subgroups to take before one can be reasonably sure that the control limits are not going to change with further samples is generally accepted at 25. According to Deming, Shewhart felt that 30 subgroups should be taken. Deming himself stated that he would like to see 60 or more subgroups before doing an analysis. These numbers are nice if one is dealing with a process that Wheeler calls "Regular Control Chart Data". However, if the data is sparse and time for decision is of the essence one is tempted to start looking at the data when 20 or more samples have been taken. The fewer the number of subgroups, the greater the chance of the control limits changing.

STABLE PROCESSES

If a control chart has been properly developed as described above, one can examine the newly measured process for assignable or special causes. If such causes are detected they must be removed. It is possible that such a special cause might occur only once through some strange circumstance and never recur. In such a case one must identify the cause and be certain that it cannot recur. One needs to be very cautious not to be misled in the fact that there will be no recurrence.

If the special cause could return, one should examine the process and take steps that the special cause will not harm the process when the special cause reappears. This might require a redesign of the process or some other safeguards.

Note that not all assignable or special causes are harmful to the process. Some special causes—such as a reduction in error rate—are desirable. In such a case, the process owner needs to consider how to change the process to incorporate the special cause as a continual component of the process.

Once harmful special causes have been removed or neutralized, what remains of the process is considered stable. The process is in control in Shewhart's sense in that it is predictable. The cost of such a process can be determined and improvements become possible. The control chart limits can then be extended into the future to monitor the process. The limits used are those computed from data that does not include any known assignable or special cause of variation. Since application of a control chart costs money for the data gathering, analysis and interpretation of the chart, one might consider reducing the monitoring system to save some of the cost without giving up the protection. This takes a form of reduced sampling.

REDUCED SAMPLING

The stable process that meets the customer's needs requires less monitoring than the process still in flux. It is essentially a tracking system that gives early warning of a departure from the normal. The work that has gone into getting to this point has a valuable data file that can be used to make decisions for reducing the tracking mechanism. This reduction only makes sense if the process is stable and the tracking expensive. If there is no cost involved with the tracking system it would be foolish to reduce it.

There are essentially two ways that the cost of the tracking system can be reduced:

- Lengthen the time between samples, and/or
- Reduce the subgroup size

Time between samples

Lengthening the time between samples reduces the number of samples taken. From the initial study one can get an estimate of the frequency and characteristics of assignable or special causes. For example, in a paper conversion process (making corrugated paper) it was observed that problems occurred when a new roll of paper was stitched to the old roll. The frequency was then set to each roll change. Additional study showed that manufacturers' lots were more closely related to problems that roll change. The frequency could be reduced even more by limiting it to the change in lots.

Knowledge of the substantive expert helps in making a better determination of frequency.

Smaller samples

Another method of reduction is to lower the already low sample size of each subgroup. Such a move will change the control limits. To reduce the number of sample in a subgroup requires modifying the control limits with these steps:

For Charts of Averages and Ranges

1. Determine that the process is stable
2. Estimate the individual standard deviation (sigma hat) from the average range $\left(\hat{\sigma} = \overline{R}_{(old)} / d_2\right)$
3. Use the $\hat{\sigma}$ to estimate the range of the new subgroup size $\left(\overline{R}_{new} = \hat{\sigma} d_{2(new)}\right)$
4. Compute new limits for the mean $\left.\begin{matrix} UCL_{(new)} \\ LCL_{(new)} \end{matrix}\right\} = \overline{\overline{X}} \pm A_{2(new)} \overline{R}_{(new)}$ where "new" is the revised subgroup size.
5. It is clear that since $\left(\overline{R}_{new} = \hat{\sigma} d_{2(new)}\right)$ and $\left(\hat{\sigma} = \overline{R}_{(old)} / d_2\right)$ that $\left(\overline{R}_{new} = \frac{d_{2(new)}}{d_{2(old)}} \overline{R}_{old}\right)$, therefore, $\left.\begin{matrix} UCL_{(new)} \\ LCL_{(new)} \end{matrix}\right\} = \overline{\overline{X}} \pm A_{2(new)} \frac{d_{2(new)}}{d_{2(old)}} \overline{R}_{old} = \overline{\overline{X}} \pm K_2 \overline{R}_{old}$. The new constant $K_2 = A_{2(new)} \frac{d_{2(new)}}{d_{2(old)}}$.
6. Compute new limits for the range as $UCL = D_{4(new)} \overline{R}_{new}$ and $LCL = D_{3(new)} \overline{R}_{new}$. Since $\left(\overline{R}_{new} = \frac{d_{2(new)}}{d_{2(old)}} \overline{R}_{old}\right)$ one can combine into a single factor the new D_4 and D_3 and the ratio $\frac{d_{2(new)}}{d_{2(old)}}$ so that the Upper and Lower Control Limits can be calculated from $\overline{R}_{old}$ to avoid all the manipulation, which could also cause computational error. Since the subscript 2 is used for the averages we propose the same subscript with a factor L_2 for the lower limit conversion and U_2 for the upper limit conversion of the limits for the range.

Hence the factors for the original and reduced sample size for means and ranges are given by:

Factor for	Subgroup Size Original	Subgroup Size, New
Averages	A_2	K_2
LCL of Range	D_3	L_2
UCL of Range	D_4	U_2

For Sample Standard Deviation

1. Determine that the process is stable
2. Estimate the individual standard deviation (sigma hat) from the average range $\left(\hat{\sigma} = \overline{s}_{(old)} / c_4\right)$
3. Use the $\hat{\sigma}$ to estimate the range of the new subgroup size $\left(\overline{s}_{new} = \hat{\sigma} c_{4(new)}\right)$
4. Compute new limits $\left.\begin{matrix} UCL_{(new)} \\ LCL_{(new)} \end{matrix}\right\} = \overline{\overline{X}} \pm A_{3(new)} \overline{s}_{(new)}$ where "new" is the revised subgroup size.
5. It is clear that since $\left(\overline{s}_{new} = \hat{\sigma} c_{4(new)}\right)$ and $\left(\hat{\sigma} = \overline{s}_{(old)} / c_4\right)$ that $\left(\overline{s}_{new} = \dfrac{c_{4(new)}}{c_{4(old)}} \overline{s}_{old}\right)$, therefore,

   $\left.\begin{matrix} UCL_{(new)} \\ LCL_{(new)} \end{matrix}\right\} = \overline{\overline{X}} \pm A_{3(new)} \dfrac{c_{4(new)}}{c_{4(old)}} \overline{s}_{old} = \overline{\overline{X}} \pm K_3 \overline{s}_{old}$. The new constant $K_3 = A_{3(new)} \dfrac{c_{4(new)}}{c_{4(old)}}$.
6. Compute new limits for the Standard Deviation as $UCL = B_{4(new)} \overline{s}_{new}$ and $LCL = B_{3(new)} \overline{s}_{new}$. Since $\left(\overline{s}_{new} = \dfrac{c_{4(new)}}{c_{4(old)}} \overline{s}_{old}\right)$ one can combine into a single factor the new B_4 and B_3 and the ratio $\dfrac{c_{4(new)}}{c_{4(old)}}$ so that the Upper and Lower Control Limits can be calculated from $\overline{s}_{old}$ to avoid all the manipulation, which could also cause computational error. Since the subscript 3 is used for the averages we propose the same subscript with a factor L_3 for the lower limit conversion and U_3 for the upper limit conversion of the limits for the range.

Hence the factors for the original and reduced sample size for means and standard deviation are given by:

Factor for	*Subgroup Size Original*	*Subgroup Size, New*
Averages	A_3	K_3
LCL of Standard Deviation	B_3	L_3
UCL of Standard Deviation	B_4	U_3

TABLES

The tables for K_2, L_2, and U_2 and for K_3, L_3, and U_3 have been computed for n = 2(1)25. An extract of the first 15 values—the most frequently used—are reproduced here. To use the tables for reduced sampling, enter old subgroup sample size on the left and read across to the column with the new subgroup sample size. The original factor is shown in bold and framed at the cell where the old size and new size intersect. The column next to the subgroup size has the factor d_2 for the range and c_4 for the standard deviation.

For example a process using subgroups of n = 5 with a mean $\overline{\overline{X}} = 9.0$ and whose range is $\overline{R} = 2.0$ had limits of $\left.\begin{matrix} UCL \\ LCL \end{matrix}\right\} = \overline{\overline{X}} \pm A_2\overline{R} = 9.0 \pm 0.577(2) = \left\{\begin{matrix} 10.154 \\ 7.846 \end{matrix}\right.$ for the chart for Averages while the limits for the Range chart are $UCL = D_4\overline{R} = 2.115 \times 2 = 4.23$ and $LCL = D_3\overline{R} = 0 \times 2 = 0$.

Table 2. Factor K_2 for Means Control Limits Using Ranges

			New Size		Reduced Subgroup Factor is unshaded											
	n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
		d2	1.128	1.693	2.059	2.326	2.534	2.704	2.847	2.970	3.078	3.173	3.258	3.336	3.407	3.472
O	2	1.128	**1.880**	1.535	1.329	1.189	1.085	1.005	0.940	0.886	0.841	0.802	0.767	0.737	0.711	0.686
r	3	1.693	1.253	**1.023**	0.886	0.793	0.724	0.670	0.627	0.591	0.560	0.534	0.512	0.492	0.474	0.458
i	4	2.059	1.030	0.841	**0.729**	0.652	0.595	0.551	0.515	0.486	0.461	0.439	0.421	0.404	0.389	0.376
g	5	2.326	0.912	0.745	0.645	**0.577**	0.527	0.488	0.456	0.430	0.408	0.389	0.372	0.358	0.345	0.333
i	6	2.534	0.837	0.683	0.592	0.529	**0.483**	0.447	0.419	0.395	0.374	0.357	0.342	0.328	0.316	0.306
n	7	2.704	0.784	0.640	0.555	0.496	0.453	**0.419**	0.392	0.370	0.351	0.334	0.320	0.308	0.296	0.286
a	8	2.847	0.745	0.608	0.527	0.471	0.430	0.398	**0.373**	0.351	0.333	0.318	0.304	0.292	0.282	0.272
l	9	2.970	0.714	0.583	0.505	0.452	0.412	0.382	0.357	**0.337**	0.319	0.305	0.292	0.280	0.270	0.261
	10	3.078	0.689	0.563	0.487	0.436	0.398	0.368	0.345	0.325	**0.308**	0.294	0.281	0.270	0.261	0.252
S	11	3.173	0.669	0.546	0.473	0.423	0.386	0.357	0.334	0.315	0.299	**0.285**	0.273	0.262	0.253	0.244
i	12	3.258	0.651	0.532	0.460	0.412	0.376	0.348	0.326	0.307	0.291	0.278	**0.266**	0.255	0.246	0.238
z	13	3.336	0.636	0.519	0.450	0.402	0.367	0.340	0.318	0.300	0.284	0.271	0.260	**0.249**	0.240	0.232
e	14	3.407	0.623	0.508	0.440	0.394	0.360	0.333	0.311	0.294	0.278	0.266	0.254	0.244	**0.235**	0.227
	15	3.472	0.611	0.499	0.432	0.386	0.353	0.327	0.306	0.288	0.273	0.261	0.249	0.240	0.231	**0.223**

Changing the subgroup size to n = 2 changes the control limits to some extent as follows:

The limits for the chart for Average are $\left.\begin{matrix} UCL_{(new)} \\ LCL_{(new)} \end{matrix}\right\} = \overline{\overline{X}} \pm K_2\overline{R}_{old} = 9.0 \pm 0.912(2.0) = \left\{\begin{matrix} 10.8234 \\ 7.1766 \end{matrix}\right.$ while the limits of the chart for Ranges are now $UCL = U_2\overline{R} = 1.5847 \times 2 \cong 3.169$ while LCL = 0.

Table 1. Factor U_2 for UCL for Range

			New Size		Reduced Subgroup Size Factor is unshaded											
	n		**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
		d2	1.128	1.693	2.059	2.326	2.534	2.704	2.847	2.970	3.078	3.173	3.258	3.336	3.407	3.472
O	**2**	1.128	**3.267**	3.862	4.164	4.359	4.501	4.612	4.703	4.780	4.847	4.905	4.958	5.005	5.048	5.087
r	**3**	1.693	2.178	**2.575**	2.776	2.906	3.000	3.075	3.135	3.187	3.231	3.270	3.305	3.336	3.365	3.392
i	**4**	2.059	1.790	2.117	**2.282**	2.389	2.467	2.528	2.578	2.620	2.656	2.688	2.717	2.743	2.767	2.788
g	**5**	2.326	1.585	1.874	2.020	**2.115**	2.183	2.237	2.281	2.319	2.351	2.380	2.405	2.428	2.449	2.468
i	**6**	2.534	1.454	1.719	1.854	1.941	**2.004**	2.053	2.094	2.128	2.158	2.184	2.207	2.228	2.247	2.265
n	**7**	2.704	1.363	1.611	1.737	1.819	1.878	**1.924**	1.962	1.994	2.022	2.047	2.068	2.088	2.106	2.123
a	**8**	2.847	1.295	1.531	1.650	1.727	1.784	1.828	**1.864**	1.894	1.921	1.944	1.965	1.983	2.000	2.016
l	**9**	2.970	1.241	1.467	1.582	1.656	1.710	1.752	1.787	**1.816**	1.841	1.864	1.883	1.901	1.918	1.933
	10	3.078	1.198	1.416	1.527	1.598	1.650	1.691	1.724	1.753	**1.777**	1.798	1.818	1.835	1.851	1.865
S	**11**	3.173	1.162	1.373	1.481	1.550	1.601	1.640	1.672	1.700	1.724	**1.744**	1.763	1.780	1.795	1.809
i	**12**	3.258	1.131	1.337	1.442	1.509	1.559	1.597	1.629	1.655	1.678	1.699	**1.717**	1.733	1.748	1.762
z	**13**	3.336	1.105	1.306	1.408	1.474	1.522	1.560	1.591	1.617	1.639	1.659	1.677	**1.693**	1.707	1.721
e	**14**	3.407	1.082	1.279	1.379	1.444	1.491	1.528	1.558	1.583	1.605	1.625	1.642	1.658	**1.672**	1.685
	15	3.472	1.062	1.255	1.353	1.417	1.463	1.499	1.528	1.553	1.575	1.594	1.611	1.627	1.641	**1.653**

Table 3. Factor L_2 for LCL for Range

New Size Reduced Subgroup Factor is unshaded

	n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
		d2	1.128	1.693	2.059	2.326	2.534	2.704	2.847	2.970	3.078	3.173	3.258	3.336	3.407	3.472
O	**2**	1.128	---	---	---	---	---	0.181	0.344	0.484	0.608	0.719	0.818	0.908	0.991	1.066
r	**3**	1.693	---	---	---	---	---	0.121	0.229	0.323	0.405	0.479	0.545	0.605	0.660	0.711
i	**4**	2.059	---	---	---	---	---	0.099	0.188	0.266	0.333	0.394	0.448	0.498	0.543	0.584
g	**5**	2.326	---	---	---	---	---	0.088	0.167	0.235	0.295	0.349	0.397	0.441	0.481	0.517
i	**6**	2.534	---	---	---	---	---	0.081	0.153	0.216	0.271	0.320	0.364	0.404	0.441	0.475
n	**7**	2.704	---	---	---	---	---	**0.076**	0.143	0.202	0.254	0.300	0.341	0.379	0.413	0.445
a	**8**	2.847	---	---	---	---	---	0.072	**0.136**	0.192	0.241	0.285	0.324	0.360	0.393	0.423
l	**9**	2.970	---	---	---	---	---	0.069	0.131	**0.184**	0.231	0.273	0.311	0.345	0.376	0.405
	10	3.078	---	---	---	---	---	0.067	0.126	0.178	**0.223**	0.264	0.300	0.333	0.363	0.391
S	**11**	3.173	---	---	---	---	---	0.065	0.122	0.172	0.216	**0.256**	0.291	0.323	0.352	0.379
i	**12**	3.258	---	---	---	---	---	0.063	0.119	0.168	0.211	0.249	**0.283**	0.314	0.343	0.369
z	**13**	3.336	---	---	---	---	---	0.061	0.116	0.164	0.206	0.243	0.277	**0.307**	0.335	0.361
e	**14**	3.407	---	---	---	---	---	0.060	0.114	0.160	0.201	0.238	0.271	0.301	**0.328**	0.353
	15	3.472	---	---	---	---	---	0.059	0.112	0.157	0.198	0.234	0.266	0.295	0.322	**0.347**

Since the lower limit of the range chart for subgroup size of 6 or less is negative, it is indicated by three dashes. In practice, the lower limit is considered to be zero in this case.

Tables for Averages and Standard Deviation

As with the lower limit for the range, the lower limit of the standard deviation chart for subgroup size

Table 4. Factor K_3 for Means Control Limits Using Standard Deviation

New Size Reduced Subgroup Sixe Factor is unshaded

	n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
		c4	0.798	0.886	0.921	0.940	0.952	0.959	0.965	0.969	0.973	0.975	0.978	0.979	0.981	0.982
O	**2**	0.798	**2.659**	2.171	1.880	1.681	1.535	1.421	1.329	1.253	1.189	1.134	1.085	1.043	1.005	0.971
r	**3**	0.886	2.394	**1.954**	1.693	1.514	1.382	1.279	1.197	1.128	1.070	1.021	0.977	0.939	0.905	0.874
i	**4**	0.921	2.302	1.880	**1.628**	1.456	1.329	1.231	1.151	1.085	1.030	0.982	0.940	0.903	0.870	0.841
g	**5**	0.940	2.257	1.843	1.596	**1.427**	1.303	1.206	1.128	1.064	1.009	0.962	0.921	0.885	0.853	0.824
i	**6**	0.952	2.229	1.820	1.576	1.410	**1.287**	1.192	1.115	1.051	0.997	0.951	0.910	0.874	0.843	0.814
n	**7**	0.959	2.211	1.805	1.564	1.398	1.277	**1.182**	1.106	1.042	0.989	0.943	0.903	0.867	0.836	0.807
a	**8**	0.965	2.198	1.795	1.554	1.390	1.269	1.175	**1.099**	1.036	0.983	0.937	0.897	0.862	0.831	0.803
l	**9**	0.969	2.188	1.787	1.547	1.384	1.264	1.170	1.094	**1.032**	0.979	0.933	0.893	0.858	0.827	0.799
	10	0.973	2.181	1.781	1.542	1.379	1.259	1.166	1.090	1.028	**0.975**	0.930	0.890	0.855	0.824	0.796
S	**11**	0.975	2.175	1.776	1.538	1.376	1.256	1.163	1.087	1.025	0.973	**0.927**	0.888	0.853	0.822	0.794
i	**12**	0.978	2.170	1.772	1.534	1.372	1.253	1.160	1.085	1.023	0.970	0.925	**0.886**	0.851	0.820	0.792
z	**13**	0.979	2.166	1.768	1.532	1.370	1.250	1.158	1.083	1.021	0.969	0.924	0.884	**0.850**	0.819	0.791
e	**14**	0.981	2.162	1.766	1.529	1.368	1.249	1.156	1.081	1.019	0.967	0.922	0.883	0.848	**0.817**	0.790
	15	0.982	2.160	1.763	1.527	1.366	1.247	1.154	1.080	1.018	0.966	0.921	0.882	0.847	0.816	**0.789**

Table 6. Factor U_3 for UCL for Standard Deviation

New Size Reduced Subgroup Size Factor is unshaded

	n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
		c4	0.798	0.886	0.921	0.940	0.952	0.959	0.965	0.969	0.973	0.975	0.978	0.979	0.981	0.982
O	2	0.798	**3.267**	2.853	2.617	2.461	2.349	2.263	2.195	2.139	2.092	2.052	2.017	1.987	1.959	1.935
r	3	0.886	2.941	**2.568**	2.356	2.216	2.115	2.038	1.976	1.926	1.884	1.848	1.816	1.789	1.764	1.742
i	4	0.921	2.829	2.470	**2.266**	2.131	2.034	1.960	1.901	1.853	1.812	1.777	1.747	1.720	1.697	1.676
g	5	0.940	2.773	2.421	2.221	**2.089**	1.994	1.921	1.863	1.816	1.776	1.742	1.712	1.686	1.663	1.643
i	6	0.952	2.739	2.392	2.194	2.064	**1.970**	1.898	1.841	1.794	1.754	1.721	1.692	1.666	1.643	1.623
n	7	0.959	2.717	2.372	2.176	2.047	1.954	**1.882**	1.826	1.779	1.740	1.707	1.678	1.652	1.630	1.609
a	8	0.965	2.701	2.358	2.163	2.035	1.942	1.871	**1.815**	1.769	1.730	1.697	1.668	1.643	1.620	1.600
l	9	0.969	2.689	2.348	2.154	2.026	1.934	1.863	1.807	**1.761**	1.722	1.689	1.661	1.635	1.613	1.593
	10	0.973	2.680	2.340	2.146	2.019	1.927	1.857	1.801	1.755	**1.716**	1.683	1.655	1.630	1.607	1.587
S	11	0.975	2.672	2.334	2.141	2.013	1.922	1.851	1.796	1.750	1.712	**1.679**	1.650	1.625	1.603	1.583
i	12	0.978	2.666	2.328	2.136	2.009	1.917	1.847	1.792	1.746	1.708	1.675	**1.646**	1.621	1.599	1.579
z	13	0.979	2.661	2.324	2.132	2.005	1.914	1.844	1.788	1.743	1.704	1.672	1.643	**1.618**	1.596	1.576
e	14	0.981	2.657	2.320	2.128	2.002	1.911	1.841	1.785	1.740	1.702	1.669	1.641	1.616	**1.594**	1.574
	15	0.982	2.653	2.317	2.125	1.999	1.908	1.838	1.783	1.738	1.699	1.667	1.639	1.614	1.592	**1.572**

Table 5. Factor L_3 for LCL for Standard Deviation

New Size New Factor is unshaded

	n		2	3	4	5	6	7	8	9	10	11	12	13	14	15
		c4	0.798	0.886	0.921	0.940	0.952	0.959	0.965	0.969	0.973	0.975	0.978	0.979	0.981	0.982
O	2	0.798	---	---	---	---	0.036	0.142	0.224	0.291	0.346	0.393	0.433	0.468	0.499	0.527
r	3	0.886	---	---	---	---	0.033	0.127	0.202	0.262	0.311	0.354	0.390	0.422	0.450	0.475
i	4	0.921	---	---	---	---	0.031	0.123	0.194	0.252	0.300	0.340	0.375	0.406	0.433	0.457
g	5	0.940	---	---	---	---	0.031	0.120	0.190	0.247	0.294	0.333	0.368	0.398	0.424	0.447
i	6	0.952	---	---	---	---	**0.030**	0.119	0.188	0.244	0.290	0.329	0.363	0.393	0.419	0.442
n	7	0.959	---	---	---	---	0.030	**0.118**	0.186	0.242	0.288	0.327	0.360	0.390	0.415	0.438
a	8	0.965	---	---	---	---	0.030	0.117	**0.185**	0.240	0.286	0.325	0.358	0.387	0.413	0.436
l	9	0.969	---	---	---	---	0.030	0.116	0.184	**0.239**	0.285	0.323	0.357	0.386	0.411	0.434
	10	0.973	---	---	---	---	0.030	0.116	0.184	0.238	**0.284**	0.322	0.355	0.384	0.410	0.432
S	11	0.975	---	---	---	---	0.030	0.116	0.183	0.238	0.283	**0.321**	0.354	0.383	0.409	0.431
i	12	0.978	---	---	---	---	0.030	0.115	0.183	0.237	0.282	0.321	**0.354**	0.382	0.408	0.430
z	13	0.979	---	---	---	---	0.030	0.115	0.182	0.237	0.282	0.320	0.353	**0.382**	0.407	0.429
e	14	0.981	---	---	---	---	0.029	0.115	0.182	0.236	0.281	0.319	0.352	0.381	**0.406**	0.429
	15	0.982	---	---	---	---	0.029	0.115	0.182	0.236	0.281	0.319	0.352	0.380	0.406	**0.428**

of 5 or less is negative, it is indicated by three dashes. In practice, the lower limit is considered to be zero in this case.

EXAMPLE

A part was measured with subgroup of n = 5. The average of the averages of 25 subgroup $\bar{\bar{X}} = 8.923$ with an average range of $\bar{R} = 1.920$ (see figure 3). Changing the subgroup size from 5 to 2 results in the following computations: $\left.\begin{matrix} UCL_{\bar{X}} \\ LCL_{\bar{X}} \end{matrix}\right\} = \bar{\bar{X}} \pm K_2\bar{R} = 8.923 \pm 0.912(1.920) = \left\{\begin{matrix} 10.674 \\ 7.172 \end{matrix}\right.$ the factor 0.912 was obtained from Table 1 above. The lower limit of the range chart does not change; it is zero since the subgroup size is less than 7 in both cases. The upper limit becomes $UCL_{\bar{R}} = U_2\bar{R} = 1.585(1.920) = 3.043$. The range needs to be adjusted to reflect the reduced subgroup size. The is done by taking the ratio of the new factor d_2 for subgroup size 2 divided by the old factor d_2 for subgroup size 5 and multiplying the this by the old range value as follows: $\overline{R}_{new} = \frac{d_{2(new)}}{d_{2(old)}} \overline{R}_{old} = \frac{1.128}{2.326}(1.920) = 0.931$. The factors d_2 were taken from Table 1 above. The ongoing process with reduced subgroup sizes is shown in figure 4.

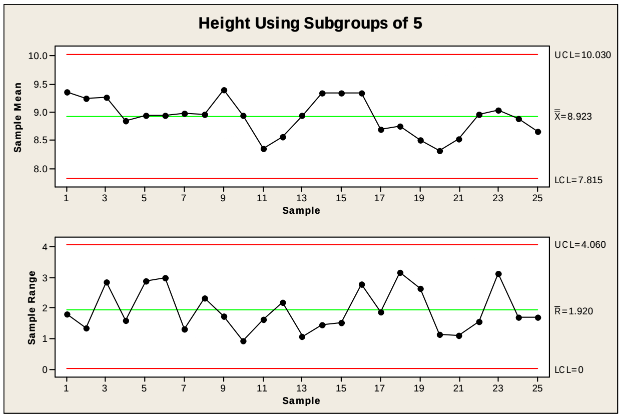

Figure 3. Bottle Height Subgroup Size 5

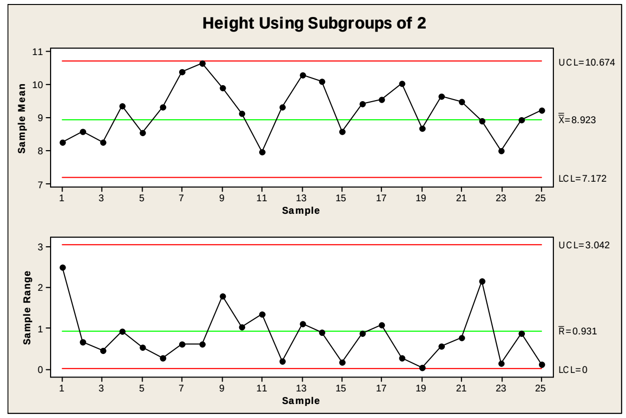

Figure 4. Height with Reduced Subgroup Size 2

Tables 4 through 6 are used in a similar manner when the measure of variability is the standard deviation rather than the range.

CONCLUSION

If the control chart is properly set up, and all assignable or special causes are removed or shown not to recur, then an ongoing chart can use the control limits that were established in the original chart. Since one initially uses as much in the way of observation as is economically viable, ongoing charts often do not require that degree of measurement. The expense of measurement can be reduced by either increasing the time interval between observations or reducing the subgroup size of each observation or both. Which method of cost reduction is used is dependent on the results of the initial chart.

This paper presented some considerations concerning the decision to reduce the cost of monitoring a process and published tables that reduce the computational process in reducing the subgroup size.

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